Abstract—Since the introduction of the idea of cognitive radio, various approaches towards spectrum sharing have been considered, for example, the Licensed Shared Access (LSA), which is considered in Europe, or Citizens Broadband Radio Service (CBRS) with Spectrum Access System (SAS) regulated by the US. This paper deals with the problem of coordinated resource allocation among a set of available base stations. A detailed definition of the problem is provided, followed by a discussion on the approaches described in the literature. Next, a new multi-selection (multi-choice) algorithm is proposed and discussed in detail. The main problem is divided in two subproblems, which are solved by using graph theorem and analytical description. The performance of the proposed solutions is analyzed in various scenarios. Finally, a trade-off between power allocation and frequency use is provided. All challenges identified during the investigation of the problem are presented.

Index Terms—CBRS model, interference management, spectrum allocation, coexistence management

I. INTRODUCTION

It is foreseen that in the future the static models of spectrum management and licensing schemes, i.e. license-exempt (as applied for, e.g., Industrial, Scientific, Medical, ISM, band or for Unlicensed National Information Infrastructure, U-NII, band) and exclusive use (as for regular cellular operators), will be complemented by flexible ones [1]. Although the static solutions are easy in implementation and management, it is envisaged that the introduction of spectrum flexibility is necessary to accommodate the expected traffic growth for future wireless networks. In this context, infrastructure and spectrum sharing have been widely investigated in the previous years, leading to significant achievements in this respect. This research is highly associated with the concept of cognitive radio, which has been introduced almost two decades ago, and which assumes the inclusion of some sort of artificial intelligence into the communication system [2], [3]. The key concept of a cognitive approach to spectrum utilization is to create an opportunity for flexible spectrum access based on various utilization goals or optimization criteria (such as the maximization of spectrum usage or minimization of mutual interference).

It is also assumed that for the protection of existing data transfers between the transmitter and receiver, an efficient algorithm that utilizes detailed information about the environment has to be provided, achieved through spectrum sensing [4] or received from a dedicated database [5], [6]. Recently, two approaches to flexible spectrum sharing have gained particular attention, mainly Licensed Shared Access (LSA) and Citizens Broadband Radio Service (CBRS) with an associated Spectrum Access System (SAS), denoted hereafter as CBRS-SAS. LSA was originally introduced by the European Commission (EC) [7] to respond to the interests of industry. In this model, additional licensed users are introduced. They can utilize spectrum bands used by other incumbent systems, as long as they follow the dedicated spectrum sharing license. The first frequency band being studied for LSA is 2.3-2.4 GHz in Europe. As a result of efforts made by the Conférence européenne des administrations des postes et des télécommunications (CEPT) and the European Telecommunications Standards Institute (ETSI), the sharing framework for LSA contains harmonized technical conditions, cross-border coordination, etc. From the architecture perspective, the implementation of the LSA concept assumes the presence of two components on top of the existing cellular architecture, mainly, LSA Repository and LSA Controller. Let us mention that the authorized shared access (ASA) concept also falls under the LSA umbrella [8], [9]. LSA evolution is also being developed to provide user priorities and more dynamic overall approach to access shared spectrum resources than the LSA system developed for the 2.3-2.4 GHz band can provide [10], [11].

The second one of the most popular spectrum allocation and sharing schemes is considered within the CBRS-SAS, where the three-tier sharing model is introduced by the FCC in the US for the 3550-3700 MHz band [12], [13]. Unlike LSA, it enables additional usage of both licensed and license-exempt-like spectrum, as long as the incumbents rights are protected. Two types of spectrum uses are introduced here, priority access licenses (PALs, being the licensed users operating along the rules similar to those in the LSA concept) and license-exempt-like general authorized access (GAA). The dedicated spectrum access system (SAS) controls the spectrum access for both
The main difference is that PAL users are protected from the interference caused by other PAL and GAA users, and in general the GAA users are not. However, recently, new approaches to managing interference between GAA users have been considered, mainly CBRS Alliance discusses the need for the introduction of dedicated coexistence manager [14] responsible for spectrum management among CBRS-SAS users. In this paper we deal with the proposal of several algorithms for efficient resource (frequency and power) allocation among CBRS-SAS base stations operating as GAA users in the three-layer model. In particular, the paper provides the following contributions:

- A detailed mathematical definition of the resource (power and frequency) allocation problem.
- Dedicated graph coloring implementation for spectrum allocation based on [14].
- Two extensions of the above-mentioned algorithm, which deal with the problem of aggregated interference.
- A multi-choice algorithm that can be run on the coexistence manager.
- A solution to the power optimization problem that guarantees optimal power allocation among GAA base stations for a given frequency allocation scheme.
- An analysis of the trade-off between power allocation and frequency split in case of interference limitations; mainly we have derived when it is better to reduce the transmit power and keep the assigned frequency band unchanged and when it is better to keep the transmit power unchanged and orthogonally split frequency band among base stations.

The rest of the paper is organized as follows. First, we present a detailed model of the CBRS-SAS system in Sec. III, which is followed by a detailed definition of the optimization problem for frequency and power allocation in the CBRS-SAS model for GAA users (see Sec. IV). Next, in Sec. V we present in detail four considered heuristic solutions for resource allocation among base stations, whereas in Sec. VI we discuss the power optimization procedure and the proposed multi-choice algorithm. Finally, Sec. VII provides a performance evaluation of the proposed schemes for various scenarios. Appendix A contains the analysis of the trade-off between power reduction and frequency split.

II. OVERVIEW OF CBRS-SAS TRANSMISSION SCHEME

In this section we provide a concise overview of a CBRS-SAS system, as well as the related work in the recent years.

A. CBRS-Based System

The Citizen Broadband Radio Service (CBRS) assumes a three-tier model introduced by the FCC in the US for the 3550–3700 MHz band [12], [13]. A diagram presenting the hierarchical relations between the three layers of CBRS-SAS users is depicted in Fig. 1. One can observe that the CBRS-SAS sharing model enables additional transmissions to be realized on both licensed and license-exempt bases, while protecting the incumbents rights. Following [15], [16], all incumbent systems have to be protected by the Citizens Broadband Radio Service Device (CBSD), i.e., the devices, fixed stations or network of such stations that are granted functioning in the shared frequency band. Two classes of CBSDs are defined (class A and B), which differ, i.e., in the maximum allowed transmit power. The exemplary incumbents to be protected are navy radars or other Department-of-Defence (DoD) systems, as well as Fixed Satellite Services (FSS) and so-called Grandfathered Wireless Broadband Licensees.

The CBRS-SAS model introduces additional licensed users holding so-called Priority Access Licenses (PALS). PAL users should protect the incumbent systems, but at the same time they should be entitled to protection from General Authorized Access Users (third tier of users) and other Priority Access Licensees. One may state that the PAL users have operational certainty similar to the LSA licensees. The GAA users are simply one of the CBSD users (regardless of the CBSD class), and have to protect the other transmission realized within higher tiers. Clearly, the PAL and GAA systems have to accept the interference originated from any systems operating in higher layers of the CBRS-based model.

The key component in the management of interference in this concept is the Spectrum Access System (SAS) that coordinates the spectrum usage of CBRS-compliant devices to protect the incumbents and PALS from other CBRS users. In principle, SAS is a combination of controlling functions and a database for the coordination of interference. Numerous detailed features of SAS are defined in [16], such as:

- determination and provision to CBSDs of permissible channels or frequencies, as well as maximum permissible transmit power level at their location
- registration and authentication of the identification information and location of CBSDs
- retention of information on so-called exclusion zones and protection zones
- resolving of any conflicts in using the frequency band while maintaining a stable radio frequency environment.

It is worth mentioning that the CBRS-SAS concept has adopted the Environmental Sensing Capability (ESC) for monitoring the incumbent activity which would detect the appearance of specific incumbents.

B. Related Work

The problem of efficient utilization of available resources has been investigated for many years, focusing recently on making the algorithms for resource allocation to users more and more flexible. In this context, various surveys have been published discussing the key achievements of the scientific society working in that domain, e.g., [17]–[19]. Next, in [20], the authors have surveyed various licensed spectrum sharing schemes from the point of view of their applicability by mobile network operators. An interesting discussion on the selection of the best access strategy for various spectrum bands is presented in [21]. Focusing more on the CBRS-SAS domain, one of the fundamental papers [13] investigates the perspectives of the CBRS-based system for the 3.5 GHz frequency band. Next,
Sahoo in [22] investigated two approaches (i.e., static, and one called max-min fairness) used for the fair treatment of GAA requests. This is due to the requirements expressed by FCC [16] to ensure the spectral efficiency and non-discriminatory coexistence of various CBSDs. Our approach presented in this paper extends this problem towards much more flexible resource allocation among the CBSDs - we allow not only for flexible power allocation, but also for the adjustment of the transmit power to meet interference constraints.

It is worth mentioning the recent trends investigated within the Wireless Innovation Forum (such as [23]–[25]), which deal with the ways of applying spectrum coordination between GAA users in the CBRS-SAS system. It is widely considered to apply tools from the network theory to create interference graphs and apply various graph coloring solutions. The main goal of these approaches is to deal with GAA spectrum channels to better maintain and coordinate the interference between GAA users. In our work, we follow this approach as well, however, the proposed solutions assume much higher level of flexibility given to SAS and the spectrum controller.

In the recent paper, i.e., [26], the authors have presented a design of the so-called hybrid centralized and distributed medium access control (HMAC) scheme for SAS. In their approach, the proposed self-organized spectrum sharing scheme applies the benefits of learning algorithms based on game theory and reinforcement learning. In particular, in the first approach the achievement of Nash equilibrium within a non-cooperative game is considered for interference minimization and fair spectrum sharing among CBSDs. What is worth mentioning in that context is that the authors have considered a carrier sensing mechanism known from the WiFi domain. An extended version of this work is presented [29].

The considerations of various user categories (associated with various requirements set to the minimum acceptable throughput) were presented in [30]. The proposed dynamic algorithm allocates the resource blocks among all stakeholders to achieve the minimum requirements assigned to each category. In order to achieve this goal, the fulfillment of the traffic requirements is evaluated by determining the transmission path-loss and shadowing effects. This paper discusses the CBRS-SAS approach in the context of 5G networks through carrying out experiments using a proprietary 5G, system-level simulation tool calibrated according to the 3GPP technical report 36.814.

Hardware experiments, and dedicated field trials evaluating the implementation of the CBRS-SAS model were proposed in [31], [32], as well as in an other paper of the authors of this manuscript, i.e., [33]. Finally, the problem of reliable detection of the incumbent radar systems operating in the 3.5 GHz band is discussed in [34].

III. SYSTEM MODEL

A. General Assumptions

In our work we consider the CBRS-SAS model [15] for spectrum sharing, where SAS is designed to manage PAL users within the second tier and the Coexistence Manager CxM is considered for interference management among GAA users [14]. Following the nomenclature used in the FCC directive...
There are devices defined in [15], the size of a square is set to 100 m. square subregions. Following the required accuracy of CBSD particularly the so-called CBRS configuration 7 is assumed. duplex (TDD) as assumed in the CBRS-SAS standard [14]. In our investigation. Moreover, our system model assumes that there is no spectrum leakage (ACLR is set to 0). The allocation of spectrum schemes to CBSDs is stored in an I × K binary matrix β with βi,k = 1 if the k-th spectrum allocation scheme is used by the i-th CBSD. As such, the total bandwidth Bi used by the i-th device can be calculated as Bi = Bw ∑n=1N ∑k=1K Γn,k βi,k. Each CBSD transmits with power Pi per a single 5 MHz channel. The maximal power is limited by a regulatory authority, i.e., Pr EG (see two classes of devices discussed in the prior subsection), device specification, i.e., Pmax ≤ Pr EG per 5 MHz, and by information obtained from the SAS database. The maximum power provided by the SAS database, denoted as Ps AS i,n, is specific for a given device location, i.e., (xi,yi) in Cartesian coordinates and for a given frequency channel n. It is assumed that a CBSD transmits equal power in each of active channels. As such the maximal power of the i-th CBSD is minimum out of Pmax and all Ps ASi,n for which βi,kΓn,k = 1 (true for all frequency channels active for the i-th device). Depending on the allocated power Pi, the range Ri of useful transmission of the i-th CBSD varies. The sensitivity of the receiver, equal to PS E N, i.e., -99 dBm/5 MHz in our case [15], is related with the Ri using the Friis propagation formula as

\[
PS E N = \frac{GTGR\lambda^2}{(4\pi)^2} Ri^{-\gamma} Pi = \alpha R_i^{-\gamma} Pi, \tag{1}
\]

where GT, GR and λ are the transmitter antenna gain, receiver antenna gain and wavelength, respectively. γ is the environment dependent pathloss exponent, equal to 2 for free space loss (FSL) propagation. The point (x,y) is within the useful range of the i-th CBSD (located at (xi,yi)) if

\[
\sqrt{(x_i - x)^2 + (y_i - y)^2} \leq Ri. \tag{2}
\]

In such a case, according to [15], the total interference from all other CBSD devices j ∈ {1,...,I} \ i is to be below the predefined interference level PINT, which in our case equals -99 dBm per 5 MHz. Assuming Adjacent Channel Interference Ratio (ACIR) between the k-th reception scheme and k transmission scheme is ACIRk,k the interference constraint for each point (x,y) within the useful range of the i-th CBSD
can be written as
\[ \sum_{j=1}^{I} \sum_{j \neq i} \alpha P_j \left( (x_j - x)^2 + (y_j - y)^2 \right)^{-\frac{2}{\beta}} \]  
(3)

The ACIR depends on the transmitter power spectral density PSD (including out-of-band radiation) and the receiver selectivity. In the case of OFDM transmission the method of calculation is given in [35]. Typically it will be a value in the range (0; 1) with for \( k = k \). Most importantly, this condition is to be met for each point \((x, y)\) within the useful range of the \(i\)-th CBSD. For practical considerations it is possible to define a dense enough grid to meet this interference constraint.

IV. JOINT SPECTRUM AND POWER ALLOCATION - PROBLEM STATEMENT

Our main optimization goal is to maximize the widths of the spectrum bands (thus, in consequence, available channel capacity) assigned to the CBSDs over possibly the widest geographical area (that is directly proportional to the assigned transmit power). However, as the CBSDs can belong to various operators having various requirements, the goal function can be case-specific. Examples can be the maximization of the sum of allocated powers \(P\), sum of allocated bands \(B\), or average throughput over analyzed area. Each of these definitions has some drawbacks, e.g., the maximization of the sum of allocated powers minimizes the bandwidth spanned by each device.

In general, a function \( f(P, \beta)\) is to be maximized by a proper choice of transmit powers \(P\) and spectrum utilization scheme \(\beta\). Considering the minimum set of constraints defined above, the optimization problem is
\[
\max f(P, \beta) \\
\text{s.t. } \forall i,j \neq i (x,y) : \alpha P_i ((x_i-x)^2+(y_i-y)^2)^{-\frac{2}{\beta}} \geq P_{\text{SEN}} \\
\sum_{j=1}^{I} \sum_{j \neq i} \alpha P_j \left( (x_j - x)^2 + (y_j - y)^2 \right)^{-\frac{2}{\beta}} \\
\cdot \sum_{k}^{K} \sum_{k}^{K} \text{ACIR}_{k,k} \beta_{i,k} \beta_{j,k} \leq P_{\text{INT}}. \\
\forall i,j : \alpha P_i \leq P_i \leq \min \{ P_{\text{max}}, P_{\text{SASi,n}} \} \\
\forall i,k : \beta_{i,k} \in \{0, 1\} \\
\forall i,k : \beta_{i,k} = 1. \\
\] 
\(\beta\)

The first constraint (5) refers to the interference limitation which has to be fulfilled for each CBSD in the network, i.e., the allowed interference at any point within the useful range of the \(i\)-th CBSD has to be limited. Constraint (6) describes the allowable ranges of the transmit power assigned to each CBSD dependent on the chosen spectrum utilization scheme. Finally, constraints (7) and (8), guarantee the selection of the contiguous band by each of the CBSDs.

Thus in the following part we will start with a heuristic analysis, which is later refined by the application of optimization algorithms for simplified optimization problems.

V. HEURISTIC ANALYSIS

As the joint spectrum and power allocation problem has been identified as a complicated optimization problem, in this section we provide a comprehensive analysis of the four proposed heuristic algorithms. These are by assumption not optimal, but they deal with the research problem in various ways. In the first three cases, the considered solutions rely on graph theory. Mainly, the first proposal is the realization of the approach defined in [14], where a dedicated interference graph is created and its chromatic number is used in the frequency allocation process. The two following solutions are our modifications of the first algorithm. The last algorithm considered here is the extended version of the iterative approach proposed in [36].

A. Graph Coloring and Finding the Chromatic Number

In this subsection, we rely on the general approach defined in [14]. Mainly, assuming the knowledge on the detailed location of CBSDs, the coexistence manager may create a so-called interference graph as proposed in, e.g., [23], [37]. In such a case, every CBSD will be represented in the undirected graph as a node, and there will be an edge between nodes \(i\) and \(j\) if node \(i\) introduces interference to any point within coverage radius \(R_j\) of node \(j\) higher than \(P_{\text{INT}}\) or the other way round. While the coverage radius of the \(i\)-th CBSD \(R_i\) is defined in (1) we can define an interference radius \(R_{\text{INT},i,j}\), i.e., the distance from the \(i\)-th transmitter where the received power equals the interference limit \(P_{\text{INT}}\), as
\[ P_{\text{INT}} = \alpha R_{\text{INT},i,j}^{-\gamma} P_i. \] 
(9)

Nodes \(i\) and \(j\) are interfering (connected in the graph) if the sum of \(R_{\text{INT},i}\) and \(R_{\text{INT},j}\) and \(R_i\) is greater than or equal to the distance between those CBSDs denoted as \(d_{i,j}\).

One may observe that in this case the interference graph will be a simple graph (i.e., neither loops nor multiple edges between two nodes are allowed), and thus such a graph may be represented in the form of the adjacency matrix \(A\), which is an \(I \times I\) square matrix, and whose elements equal 1 if there exists an edge between two particular nodes. It is created using (9) and (1) as
\[ A_{i,j} = \begin{cases} 1 & \text{if } \max \left( \frac{\alpha P_i}{P_{\text{INT}}} \right)^{\frac{1}{\gamma}} + \left( \frac{\alpha P_j}{P_{\text{SEN}}} \right)^{\frac{1}{\gamma}} \geq d_{i,j} \\ 0 & \text{in other cases} \end{cases} \] 
(10)
In general, three ways of removing an edge from the graph may be considered: either the transmit power of the interfering nodes will be reduced, or the locations of these nodes can be changed, or the available spectrum band can be split orthogonally between these two nodes. In our case we assume that the location of the device is fixed (i.e., even for nomadic networks, the coexistence manager can provide a suggestion about the available spectrum band for a certain location provided by the device in the query). It can be assumed that the distance between nodes $d_{i,j}$ and requirements on the sensitivity level $P_{\text{SEN}}$ and interference level $P_{\text{INT}}$ are fixed. However, the potential interference depends additionally on the allocated band (carrier frequency changes $\alpha$) and allocated power $P_i$ and $P_j$ as visible in (10). A worst-case scenario is assumed here with the minimum center frequency out of the available band, i.e., $f = 3652.5$ MHz, guaranteeing the maximum range of interference. Additionally, the worst-case power $P_{\text{WC}}$ is defined as being the maximum out of all possible frequency channels $n$, i.e.,

$$P_{\text{WC}i} = \min\{P_{\text{max}}, \max_n\{P_{\text{SASi,} n}\}\}. \quad (11)$$

The worst-case power provides both the highest interference radius and highest protected area (defined by radius $R_i$). As such, two nodes are linked in the interference graph if there is any spectrum and power allocation scheme that can cause such an interference.

In order to find the minimum number of orthogonal sets in any graph, the algorithms for node coloring and finding the so-called chromatic number can be applied. The chromatic number defines the minimum number of colors required to color each node in a given graph, assuming that always different colors are used in connected nodes. In our case, the calculated chromatic number $X$ defines the minimum number of non-overlapping frequency subbands.

In the final step, the division of the whole available bandwidth into $X$ subbands and the assignment of those subbands to colors has to be performed, which is equivalent to the assignment of frequency bands to certain CBSDs. It is not a straight-forward problem, as the presence of interference from adjacent channels has to be considered. In the worst case it will be necessary to allocate two adjacent subbands to close CBSDs, and such a situation may entail the need for the creation of a dedicated guard band, e.g., of bandwidth $B_w$.

One solution is to consider all possible combinations of color-band assignment (giving $X!$ options) and select the one that minimizes the guard band.

An exemplary interference graph and the resultant subband allocation among colors is shown in Fig. 2. One may observe the presence of 9 CBSDs, grouped into three subgraphs. The dashed circles represent the coverage area of the CBSD the assuming maximum allowable transmit power. The edges between the nodes indicate the presence of interference (if both nodes use the same band), and different colors represent different frequency subbands. One may observe that in the provided example the chromatic number is $C = 3$. On the right side of the figure one may observe the frequency allocations, i.e., the vertical axes represent CBSDs, and the horizontal one - the frequency blocks. White color in this figure means that a given frequency block has not been assigned to this certain CBSD, and the various tones of copper color define different frequency blocks. One may observe that for CBSDs with indexes $\{1, 2, 3, 4, 6\}$ the entire spectrum has been split into three orthogonal subblocks. Analogously, devices $\{5, 7, 8\}$ split the band into three bands, and CBSD 9 can use the entire band. These three groups of nodes are indicated in the right-side plot by various shades of color (i.e., light brown, brown and dark brown). As such, graph coloring and band allocation is carried out separately for each subgraph. Next, looking at the left part of the figure one may notice that nodes 5, 7 and 8 are marked with various colors, which means that three non-overlapping frequency blocks have been assigned to them. At the same time, pairs of node $\{1, 2\}$ and $\{3, 6\}$, are colored with blue and red, respectively, which indicates that the same frequency bands are assigned to each node in the group. Finally, let us notice that there was no need to introduce guard bands for nodes $\{5, 7, 8\}$. It was, however, necessary to add guard bands (one block of 5 MHz) between frequency blocks allocated to node 4 and pairs $\{1, 2\}$ and $\{3, 6\}$.

The presented solution is described in the form of an Algorithm 1.

Algorithm 1: Interference Graph Creation and Coloring

Data: CBSDs setup (location, transmit power)
Result: Colored Interference Graph and Initial Resource Allocation

1. Deploy $I$ CBSDs in the give area;
2. foreach pair $(i, j)$ of CBSD do
3.     | create an edge in the graph according to (10)
4. end
5. Once the graph is created, apply the coloring algorithm
6. and find the chromatic number $X$;
7. Assign frequency subbands to colors;
8. Add guard bands if necessary;
9. end

It relies on the approach proposed in [14] and may be the basis for further analysis and improvements, as we discuss in the following sections.

B. Aggregated Interference Problem - Heuristic Approaches

The graph coloring approach in the form presented above does not take into account the problem of aggregated in-
interference. Although the interference from another CBSD transmitting in the adjacent band is somehow considered by the application of the guard bands, it does not solve the problem of aggregated interference. The interference graph consists of edges that represent the presence of interference between two particular nodes, but does not reflect the issue of summation of the interference power originated from many, even not connected nodes. As shown in Fig. 2, there is no violation of the interference requirements between nodes 5 and 9. However, the total aggregated interference observed by node 5 appeared to be above the limit. In order to deal with this problem, we have proposed two heuristic approaches described below.

1) **Graph Coloring with Add-Edge Algorithm**: In the first case we assume that the entire interference graph will be updated in such a way that the aggregated interference (that causes problems) is also considered. In order to update the interference graph, for the violated CBSD $i$ (i.e., the node that observed too severe aggregated interference) we create a list of interferers not connected to a given node, and find the most distorting node $j$. Once it has been identified, an edge between nodes $i$ and $j$ is added to the graph, and the adjacency matrix is updated (i.e., $A_{i,j} = A_{j,i} = 1$). Thus, we call this approach an *add-edge* procedure. Next, graph coloring and band allocation as in Algorithm 1 is executed. The *add-edge* procedure is repeated as long as there are some CBSDs that observe too severe aggregated interference. The entire procedure may be summarized as in Algorithm 2. The interference graph from Fig. 2 is then modified as shown in Fig. 3, where the additional edge between nodes 5 and 9 is created. The corresponding frequency allocation among nodes 5, 7, 8 and 9 is changed as well.

2) **Graph Coloring with Reduce Power Algorithm**: In the second proposed approach the interference graph is not modified, thus the spectrum split (achieved as the outcome of Algorithm 1) is not changed. The elimination of the harmful aggregated interference is achieved here by the proper adjustment of the transmit power of interfering nodes. Similarly to Algorithm 2, the most violated node has to be detected, in the next step the most influencing node (with respect to the identified violated one) has to be identified and in consequence its transmit power has to be reduced by, e.g., 1 dB (this is an arbitrarily chosen value that can be adjusted due to various circumstances). After the power reduction step the presence of too strong aggregated interference has to be checked, and if necessary the entire loop has to be repeated one more time. The entire procedure has been summarized in Algorithm 3.

![Interference graph](image)

**Fig. 3.** Interference graph after executing *add-edge* procedure

<table>
<thead>
<tr>
<th><strong>Algorithm 2:</strong> Graph Coloring with Add Edge Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data:</strong> CBSDs setup (location, transmit power)</td>
</tr>
<tr>
<td><strong>Result:</strong> Colored Interference Graph, Resource Allocation with Acceptable Aggregated Interference</td>
</tr>
<tr>
<td>1 Apply Algorithm 1;</td>
</tr>
<tr>
<td>2 Calculate aggregated interference;</td>
</tr>
<tr>
<td>3 while <em>too severe aggregated interference in the network</em> do</td>
</tr>
<tr>
<td>4 Find strongest interferer $j$ (not connected previously) for selected most violated node $i$;</td>
</tr>
<tr>
<td>5 Add edge between nodes $i$ and $j$ in interference graph;</td>
</tr>
<tr>
<td>6 Find chromatic number $X$ and apply the coloring algorithm;</td>
</tr>
<tr>
<td>7 Assign frequency subbands to colors (including guard bands);</td>
</tr>
<tr>
<td>8 Calculate aggregated interference;</td>
</tr>
<tr>
<td>9 end</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th><strong>Algorithm 3:</strong> Graph Coloring with Reduce Power Algorithm</th>
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<tbody>
<tr>
<td><strong>Data:</strong> CBSDs setup (location, transmit power)</td>
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</tr>
<tr>
<td>4 Find strongest interferer $j$ for selected violated node $i$;</td>
</tr>
<tr>
<td>5 Reduce the transmit power $P_j$ of node $j$ by a certain value, e.g., 1 dB;</td>
</tr>
<tr>
<td>6 Update interference graph;</td>
</tr>
<tr>
<td>7 Find chromatic number $X$ and apply the coloring algorithm;</td>
</tr>
<tr>
<td>8 Assign frequency subbands to colors (including guard bands);</td>
</tr>
<tr>
<td>9 Calculate aggregated interference;</td>
</tr>
<tr>
<td>10 end</td>
</tr>
</tbody>
</table>

| **C. Iterative Approach** |

In [36] an Iterative Allocation Process (IAP) is proposed to deal with the CBSDs resource allocation. Following IAP, SAS takes iteratively all requests expressed by the CBSDs and checks if the allowable levels for aggregated interference is violated at any identified protection point or not. If for a given request the interference levels are not violated for any existing CBSD at any protection point, then such a request is accepted and requested resources are granted for this CBSD. Next, the
consecutive request is processed. In the case of violation of the allowable interference level at any of the referenced points, SAS reduces the allowable transmit power for the considered CBSD and verifies the interference levels observed at the reference points. The last steps are repeated iteratively until the interference requirements are fulfilled or when the transmit power falls below some predefined threshold.

It is worth noticing that IAP does not assume a change of the frequency bands when the interference requirement is not satisfied. In order to improve the performance of this iterative algorithm, we have slightly adjusted it to consider the availability of multiple frequency channels. First, it is assumed that the entire available bandwidth is divided between CBSDs in such a way that for each CBSD a certain predefined (equal) number of 5 MHz width, adjacent frequency blocks may be assigned from the total $N = 30$ blocks\footnote{This approach may be easily modified to a more flexible case, where each CBSD provides an individual spectrum request}. The key idea in the updated algorithm is that the coexistence manager tries to allocate the contiguous frequency band (with possible maximum power) before it makes a decision on transmit power reduction. In particular, the algorithm starts with the index of the first frequency block and checks if the particular fragment can be allocated to this CBSD with given maximum power $P_i$. If not, it increments the block index and checks the transmit opportunities again, i.e., if the interference constraints are met. If the algorithm reaches the last possible frequency block and allocation is impossible, the requested transmission power is reduced as in the original IAP algorithm and the search starts again from the first frequency block. The modified IAP algorithm is presented as Algorithm 4.

\section{Algorithm 4: Updated IAP}

\begin{algorithm}
\begin{algorithmic}[1]
\Statex \textbf{Data:} CBSDs setup (location, maximum transmit power)
\Statex \textbf{Result:} Resource Allocation with Acceptable Aggregated Interference
\Function{CBSD requesting $R \leq N$ frequency blocks}{do}
\State Set frequency block index $n = 1$;
\State Set the transmit power $P_i$ to minimum value from the vector of $R+1$ constrains as $P_i = \min \{ P_{\text{max}}, P_{\text{SAS}i,n}, \ldots, P_{\text{SAS}i,n+R-1} \}$;
\While {frequency resources are not allocated for considered $i$-th CBSD}
\If {there is no possibility to assign $R$ blocks starting from index $n \leq N - R$ due to violation of interference constraints}
\If {$n \leq N - R$}
\State Increase index $n = n + 1$
\Else
\State Reduce transmit power $P_i$ for considered CBSD by e.g. 1dB;
\State Set frequency block index $n = 1$;
\EndIf
\Else
\If {allowable transmit power is below acceptable threshold}
\State Discard this CBSD request;
\EndIf
\EndIf
\EndWhile
\EndFunction
\end{algorithmic}
\end{algorithm}

\section{VI. Optimized Power Allocation and Multi-Choice Algorithm}

The presented meta-heuristic algorithms find by default only one solution, which may not always fulfill each CBSD’s requirements. For example, if there is a requirement that the minimum reasonable bandwidth allocated to the CBSD be above some certain value, then the solutions found by the above algorithms may not be acceptable. Analogous conclusions could be drawn for the situation where the minimum radius of the cell (thus transmit power) is also defined. For this reason we have proposed the Multi-Choice Algorithm that produces $M_{\text{max}}$ solutions, where $M_{\text{max}}$ is limited either by the number of available channels or the chromatic numbers, so $M_{\text{max}} = \min (X, N)$. Such an approach will deliver the entire spectrum of solutions, starting from the situation where all nodes use the same frequency band, and ending when the available spectrum is split orthogonally among all CBSDs.

Let us observe that the problem of efficient resource allocation among CBSDs consists of two (associated) subproblems. The first one is related to the right split of the available frequency band and assignment of the resultant subbands into CBSDs. The second one deals with power optimization in each frequency band. In what follows, we will present the proposed power optimization algorithm, which will be widely applied in the proposed multi-choice algorithm once the frequency assignment has been done.

\subsection{A. Power Optimization}

It is possible to simplify the optimization problem (4) for a case of $I$ devices operating in the same band. This is a subset of all devices, i.e., $I \leq I$. It can be the result of a fixed $\beta$ matrix and choosing a common activity band. In the final algorithm, power optimization will be carried out for each frequency channel (out of $N$) separately. However, it is possible to optimize the power for a wider band as well. Fixed $\beta$ removes the requirement of constraints (7) and (8). Additionally, the upper limit of power $P_i$ is a fixed number denoted as $P_{\text{MAX}}$. It is a solution of the optimization problem $\forall i \forall n : \beta_{i,k} P_{n,k} = 1 \min \{ P_{\text{MAX}}, P_{\text{SAS}i,n} \}$. Finally, the interference constraint (5) can be simplified. It is not required to check for an exceeded interference level inside the cell (for multiband optimization interference can occur as a result of, e.g., OOB radiation from adjacent band). It is enough to check the interference on a cell border. ACIR in this case equals 1. This significantly reduces the number of constraints. The goal function is only dependent on a vector $P$. It is chosen to maximize sum of $P_i$ values weighted by power coefficient $\rho$. High $\rho$ values prioritize CBSDs transmitting with high power, while lower values promote equality between CBSDs’ powers. This is one of the implementations of the goal function $f(P, \beta)$ with constant $\beta$, chosen for practical purposes. These
simplifications allow us to define the optimization problem as

\[
\max_P \sum_i P_i^o \quad (12)
\]

subject to

\[
\begin{align*}
\forall_i, \forall_{m,\alpha} \sum_{j=1}^{\gamma_i} g_{i,j,m} P_j & \leq P_{INT} \quad (13) \\
\forall_i \ 0 \leq P_i \leq P_{MAX_i} & \\
\end{align*}
\]

where \(g_{i,j,m}\) is the distance between the \(j\)-th CBSD and \(m\)-th point on the border of the \(i\)-th CBSD. Let us define \(M\) protected points on the border of each CBSD. Each of these is defined by an angle \(\theta_m\) for \(m = 1, \ldots, M\) and cell radius \(R_i\) in polar coordinates for \(i\)-th CBSD as shown in Figure 4. The distance between \(i\)-th and \(j\)-th CBSD equals \(d_{i,j}\) and angle of vector between \(i\)-th CBSD and \(j\)-th CBSD in polar coordinates equals \(\phi_{i,j}\). Using law of cosines, the distance \(g_{i,j,m}\) can be calculated as

\[
g_{i,j,m} = \sqrt{d_{i,j}^2 + R_i^2 - 2d_{i,j}R_i \cos (\theta_m - \phi_{i,j})}. \quad (15)
\]

Observe that the above optimization problem depends only on \(P_i\) variables after substituting \(R_i\) using (1). This problem is solved by means of function \texttt{fmincon} available in Matlab.

### B. Concept of a Multi-Choice Algorithm

Once the power optimization algorithm has been discussed in detail, let us now present the entire multi-choice algorithm, which is illustrated in Fig. 5. It is split conceptually into two main phases, the first one, where Graph Coloring with Add-Edge Procedure is applied (Alg. 2), and the second, where node clustering is applied iteratively. In the first phase, only one solution is generated, i.e., the one where all devices transmit with the maximum allowable power (this is obtained by maximal band fragmentation). In the second phase, the remaining \(M_{MAX} - 1\) solutions are created. The entire algorithm is discussed in the following subsections.

1) Initialization and graph coloring: As in the previous algorithms, as the input to the algorithm we take the limitations coming from SAS and from CBSDs specifications, mainly the maximum equivalent transmit power for each device is provided. Next, as the parameter we take the acceptable interference level \(P_{INT}\). We start with the creation of an interference graph and calculation of chromatic number \(X\), as described in Sec. V-A.

2) Phase 1 - Assignment of colors to frequency bands: Next, the algorithm enters the step of assigning of frequency subbands to the colors. Currently this is realized by the generation of all \(X!\) possibilities and selection of the best one minimizing number of the required guard band. In particular, we check each combination of color pairs if they can occupy adjacent frequency bands, or if they need one channel of separation as a guard band to prevent excessive out-of-band interference. Next, we try all combinations of color pair order and search for the one that requires the minimal number of guard bands. When we know the order of colors and places for the guard bands, we only need to choose how many channels (subbands) are assigned to each color. We used two solutions. The first one assumes that every color should have the same amount of spectrum. The second one assumes assignment proportional to the number of devices utilizing a given subband, i.e., colors with a higher number of devices should have more spectrum than others. For example, if we have 3 devices associated with 2 colors, one color should get 1/3 of the spectrum and the second one 2/3 of the spectrum. The former approach is hereafter called \textit{Equal assignment}, whereas the latter - \textit{Proportional Assignment}. One may observe that the order of coloring and assignment of frequency bands to colors simply matters. Thus, it may happen that there are some channels left, for example because the number of channels (without guard bands) is not always divisible by the number of colors \(X\). In such a case we simply add these free channels to random colors (CBSDs). Observe that the above-described procedure is carried out for each subgraph (not connected to another subgraph) separately.

3) Phase 1 - Add edge procedure: In order to verify the correctness of the created graph and resultant frequency assignment, the total observed aggregated interference (denoted in the algorithm as \(P_{INT}\)) is calculated for each node on its coverage area. If the aggregated interference from many CBSDs violates the interference constraint, it has to be eliminated, and this is done by the application of the Add Edge algorithm (Algorithm 2), repeated as many times as the aggregated interference occurs (see also Sec. V-B1). Once this constraint is fulfilled, the created graph is treated as the outcome of the first phase of the Multi-Choice Algorithm and delivered as input to the second phase.

One may notice that this graph and its corresponding chromatic number guarantee interference-free transmission with the maximum power for all \(I\) CBSDs. It means that the available spectrum has to be split into \(X\) orthogonal, not necessarily equal (as mentioned in the previous subsection), subbands of the bandwidth being the integer multiplication of 5 MHz. If such a division is possible, then we treat such a spectrum assignment as the first possible solution and store it. The created solution may be, however, not possible if the number of CBSDs is large and the interference graph very dense. To be more precise, when the final chromatic number \(X\) is greater than the number of available 5 MHz channels \(N\), then it is impossible to split the spectrum as described...
earlier. In such a case, the outcome of the first phase of the algorithm is the interference graph that guarantees the lack of aggregated-interference problem. No possible solution for frequency assignment will be created then.

4) **Phase 2 - Iterative procedure:** The outcome from Phase 1 of the Multi-Choice Algorithm is, first, a network graph that may be colored in such a way that there is no aggregated interference; second, chromatic number $X$, and third, if applicable, a unique solution for the maximum transmit power. The ultimate goal of Phase 2 is to show the trade-off between the allocation of power and frequency to the CBSDs. The algorithm calculates the optimal power assignment for all potentially applicable numbers of frequency divisions. In other words, it starts with the case where all CBSDs are clustered together (i.e., they use the same frequency or equivalently the number of colors in the graph is one) and optimize the transmit power in the network, taking into account each 5 MHz band individually. If by $m$ we denote the current number of CBSD clusters, then the algorithm starts with $m = 1$. Next, the number of colors is incremented, which is equivalent to increasing the total number of CBSD clusters by one ($m = 2$). In other words, the entire band will now be split in two parts. Again, the optimal power assignment is applied. This procedure is repeated $M_{\text{max}} - 1$ times, thus as long as the number of clusters equals the chromatic number $X$ or when the number of clusters equals the number of available 5 MHz channels.
5) Phase 2 - Merging Colors: The important issue is to find the way for cluster creation, mainly, how should the CBSDs be assigned to the cluster? It is done to decrease spectrum fragmentation. The input is the colored interference graph from Phase 1. It contains $X$ various colors. In order to achieve $m$ colors, $X - m + 1$ colors have to be merged, and we propose to do this iteratively. The main idea is to merge two least interfering colors in the interference graph every time. It is achieved through the maximization of the minimum pathloss between the CBSDs of two colors to be merged. Observe that at this stage the frequency allocation obtained in Phase 1 or previous iterations of Phase 2 is not used.

Let us define matrix $W_{a,b}$ of size $C_a \times C_b$. Indexes $a$ and $b$ denote a given graph color, i.e., $a, b \in \{1, ..., X\}$. The number of CBSDs utilizing the $a-th$ color is denoted by $C_a$. A matrix element $W_{g,h}^{a,b}$ denotes the pathloss between the $g-th$ CBSD of color $a$ and $h-th$ CBSD of color $b$, where $g \in \{1, ..., C_a\}$ and $h \in \{1, ..., C_b\}$. This pathloss is calculated based on a proper element $d_{a,j}$. The following dedicated metric has been used to rate any pair of colors $a, b$:

$$w_{a,b} = \min_{g,h} W_{g,h}^{a,b}. \quad (16)$$

The result $w_{a,b}$ can be interpreted as an interference attenuation between colors $a$ and $b$. Although in total $X^2$ combinations are possible, it can be observed that $w_{a,b} \approx w_{b,a}$ and there is no point in calculating $w_{a,a}$. It is only required to do the above optimization for $\frac{X(X-1)}{2}$ pairs of colors, i.e., $a \in \{1, ..., X - 1\}$ and $b \in \{a + 1, ..., X\}$. The pair of colors to be merged $(a, b)$ is chosen based on the above set as

$$(a, b) = \arg \max_{a \in \{1, ..., X-1\}} \min_{b \in \{a+1, ..., X\}} w_{a,b}. \quad (17)$$

The pair of colors guaranteeing the maximal minimal pathloss between CBSDs is chosen (max-min problem). This procedure is repeated until the required number of clusters $m$ is obtained. Using this method we can create any cluster count from 1 to $M_{max}$.

6) Power optimization: Once the colors are merged, the next step is to assign frequency subbands to the merged colors, as described in Phase 1. After that the power optimization algorithm from Sec. VI-A is applied for each frequency channel $n$ independently. The power of the $i$-th CBSD $P_i$ is the minimum out of all power values obtained as a result of power optimization carried out for each active channel (for $i-th$ CBSD), i.e., $P_i = \min_{\forall n, \beta_i, k_i} x_{i,n,k} = 1$.

While in the first phase the algorithm generates potentially only one feasible solution (i.e., the one where all transmit powers are maximal), in the second phase one will achieve $M_{max} - 1$ solutions. The Coexistence Manager will then need to decide (based on given decision criteria) which solution is the most suitable. We discuss the performance of this algorithm, as well as various selection criteria in the following section.

VII. PERFORMANCE EVALUATION

In this section we discuss the results of computer simulations. The entire simulator has been implemented in the Matlab environment, where several toolboxes have been utilized, e.g., for solving the power optimization problem.

We start our discussion with the presentation of the results from the exhaustive search simulation and compare the allowed system configurations with the results of the proposed algorithms (Algorithms 1 to 4 and the Multi-Choice one). Finally, we discuss in detail the features of the proposed multi-choice algorithm in various scenarios.

Let us recap that in our simulation we use the system model described in detail in Sec. III, mainly, each CBSD may be allocated with a contiguous band in the range from 3.550 GHz to 3.7 GHz, split into $N = 30$ channels, where the bandwidth is set to $B_w = 5 MHz$. The interference limit $P_{INT} = -90 dBm/5 MHz$, the coverage area of each CBSDs is defined as a circle of the radius defined in (1) for the free space path loss model with $\gamma = 2$. The CBSDs are deployed over the square area of size $40 km \times 40 km$, split into $Q = 160000$ subregions of size $100m \times 100m$. Moreover, in the simulations we consider the guidelines delivered by SAS, mainly for each CBSD location a vector of size $N$ is created that contains the maximum power that can be potentially used by the $i-th$ CBSD on each frequency channel, i.e., $P_{SAS,i,n}, n = 1, ..., N$. The values of $P_{SAS,i,n}$ were chosen as a random variable uniformly distributed in the range $[-17, 44] dBm$ confirming correctness of the proposed solutions. However, the results presented below are obtained for constant $P_{SAS,i,n}$ values both over the frequency range and CBSD index. It simplifies the interpretation of the results. This removes randomness caused by SAS. If not stated differently, we have considered CBSD from Class B, i.e., the maximum power defined in [15] cannot exceed $P_{MAX} = 44 dBm$ measured in the 5 MHz band. It is assumed the CBSDs TX and RX use omnidirectional antennas of gains $G_{TX} = 1$ and $G_{RX} = 1$. Power optimization utilizes $\rho = 0.5$, providing a relatively high equality of assigned powers between CBSDs.

A. Exhaustive Search and Solution Space

In order to visualize the space of feasible solutions, we have applied the brute force algorithm. For the defined number of frequency channels $N$ and the number of possible transmit power levels $N_p$, considering all possible locations of the assigned spectrum blocks in the available range, i.e., $(N + 1)N/2$ as shown previously, we calculate the value of corresponding interference and aggregated interference. The total number of system configurations is $(1 + N)^N N_p$. If for the given setup there is no violation of any of the assumed constraints, such a setup is included into the space of available solutions, otherwise it is marked as prohibited setup.

As the total number of options significantly increases with the increase of the number of CBSDs, available channels and power range as shown above, we have decided to show the brute force results for a simplified scenario. Such a simplification, however, has no impact on the overall conclusion, which can be drawn. In this subsection we have assumed the presence of $I = 3$ CBSDs, which may use up to $N = 5$ available frequency channels with $N_p = 7$ steps of transmission power (from $10 dBm$ to $40 dBm$, step $5 dBm$). Two location options showed in Figures 6 and 7 were considered.
In Figures 6–11, the red dots represents all tested setups for frequency and power assignment not meeting interference constraints, whereas the black dots constitute the domain of feasible solutions. One may notice that the space of available solutions is discrete, as the input data for the brute force algorithms are also discrete. Moreover, on the horizontal axis the sum of allocated bandwidths for all CBSDs, i.e., $\sum_{i=1}^{I} B_i$ is considered, and on the vertical axis the mean PSD is shown, i.e., $\frac{1}{I} \sum_{i=1}^{I} P_i$. It may be concluded that the points generated in the brute force algorithm may overlap (regardless of the color of the nodes). The same mean PSD and total bandwidth may be achieved for various setups with some of them obeying constraints and other not.

Solution spaces in these cases are showed in Figures 8 and 9. One issue to be discussed is the presence of black tails in the horizontal direction, a sequence of black dots surrounded by red dots. An immediate question may appear how it is possible that the point, e.g., 35 dBm per 5MHz for 35 MHz is acceptable, and 33 dBm (for the same summarized bandwidth) is not allowed? The reason is the fact that this result has been achieved for a discrete set of power values, i.e., value 33 came from averaging, e.g., two high power values and one close to zero, and this solution may not be possible.

In general, it is visible that it is impossible to provide a maximum bandwidth, i.e., 25 MHz while keeping the maximum transmission power, i.e., 40 dBm, in each device (upper right corner in Fig. 8 and 9) as a result of interference constraints not met. On the other hand, results in the lower left corner, providing a minimum bandwidth and transmit power, are acceptable from the constraints perspective. However, they provide low spectral efficiency. Depending on the construction of the goal function $f(P, \beta)$ the optimal solution will differ, however, it is expected that it will be located close to the border between red and black dots areas.

### B. Performance Comparison

Let us now compare the performance of the above proposed algorithms. It can be based on the solution set presented above. However, as proposed in Sec. IV, the maximization of the mean throughput over the whole area can be used as a goal function. Let us define the summarized rate for the $i$-th CBSD
over its cell area as
\[
S_i = \int \int_{A_i} B_i \log_2 \left( 1 + \frac{\alpha P_i \left( (x_i-x)^2 + (y_i-y)^2 \right)^{-1}}{N + \sum_{j \neq i} \alpha P_j \left( (x_j-x)^2 + (y_j-y)^2 \right)^{-1} \sum_{k} K \sum_{j,k} \text{ACIR}_{k,j,k} + \beta_{i,j,k}} \right)
\]
where \( N \) is thermal noise power over the 5 MHz band and integration area is
\[
A_i = \{(x,y) : (x,y) \neq (x_i,y_i) \land \alpha P_i \left( (x-x_i)^2 + (y-y_i)^2 \right)^{-1} \geq P_{\text{SEN}} \}.
\]
Observe that the point coinciding with the i-th CBSD location is excluded from calculation, as the FSL propagation model works erroneously in this point, i.e., causes infinite received power for \((x,y) = (x_i,y_i)\). Additionally, formula (18) is simplified in the frequency domain. It considers neither frequency selective channel fading, nor interference changing between 5 MHz blocks. The mean interference in the whole band is considered. As the number of figures is relatively high (it is done intentionally to reflect in detail the behaviour of the algorithm), we split the description of them into two parts.

1) Figures from 12 to 18: For the single, exemplary deployment of CBSDs (shown, e.g., in Fig. 12), the calculated chromatic number equals \( X = 5 \), thus the 5 steps of the Multi-Choice Algorithm are shown in Figures 12 (for five clusters, each cluster utilizing a separate band) to Figure 16 (for one cluster of CBSDs). In each figure one may observe the coverage area of each CBSD, the corresponding interference graph which has been already colored, and the final split of frequencies among devices. When necessary, the algorithm adds the one-channel frequency gap (guard band) between two adjacent blocks of frequencies.

Furthermore, in order to obtain statistically meaningful results, the Multi-Choice Algorithm was run for 100 random deployments of 10 CBSDs. In Figure 17, Empirical Cumulative Density Functions (ECDFs) of allocated bandwidth and power are shown for the changing number of CBSD clusters from 1 to 10. Expectedly, for a low number of clusters, high bandwidth is assigned (e.g., 150 MHz for each device for 1 cluster) at the cost of reduced transmit power (e.g., about 80% of CBSDs transmit with power below 20 dBm for 1 cluster). For a high number of clusters, high transmission power is possible but with significantly reduced bandwidth.

The proposed Multi-Choice Algorithm allows a single setup to be chosen based on a specific goal function or minimum requirements of each operator/CBSD.

As an example rate averaged over the whole simulation area and all \( I = 10 \) CBSDs is calculated as defined in (19). The minimum, mean and maximal values (over 100 random runs) are plotted in Figure 18. One may observe that the highest rate is achieved for the highest number of clusters. Maximal band partitioning is optimal from the perspective of this metric and uniformly distributed CBSDs. However, for different CBSD distributions this can change. An analytical derivation of the mean area rate for the case of two base stations utilizing the same or orthogonal frequency bands is shown in Appendix A. The conclusion is that for two CBSDs
located close to each other (distance smaller than about 0.7·2 of the maximum cell radius obtained for maximum allowed power) it is advantageous to use a narrower but orthogonal band. However, for wider distances it is advantageous to reduce the transmission power, but use the whole bandwidth in each CBSD. This can be the case encountered in a real network where base stations are significantly distanced in order to increase coverage. Overlapping is typically used only for handover purposes.

2) Figures from 19 to 23: An example of such a deployment (i.e., the one where it will be beneficial to reduce the transmit power) is shown in Fig. 19. All CBSDs can transmit with the maximum power of 16 dBm/5 MHz resulting in the maximum cell radius 3.7 km. The distance between two closest CBSDs equals 5 km. This is about 0.68·2 of the maximal cell radius, i.e., close to the border value obtained in Appendix A. The grid-like deployment of CBSDs resulted in a densely connected graph and the chromatic number equals $X = 4$.

Figures 19-22 show results of the graph coloring, cell areas and band distribution for the number of clusters decreasing from 4 to 1, respectively. The mean area rate calculated according to (19) is shown in Figure 23. The maximization of this metric is obtained for two clusters of CBSDs. This is in line with the results obtained in Appendix A.

Finally, let us compare the performance of the Multi-Choice Algorithm when the traditional and proportional approach to channel allocation among CBSDs is considered. Let us remind that in the former case the set of available resources is split equally between CBSDs, whereas in the latter case, the amount of frequency resources is directly proportional to the number of CBSDs in each cluster. The comparison of these two approaches is shown in Figs. 25-26. As it could be expected, both algorithms guarantee the same performance for the minimum number of clusters (i.e., all CBSDs are in the same cluster) and for the maximum number (i.e., the number of colors equals the number of CBSDs, $X = N$). In all other cases the mean rate per one squared kilometer is higher for the traditional approach, but the span between the maximum and minimum rate is high. In the proportional case, this span is much narrower, i.e., the distribution is much fairer. The corresponding frequency allocation among CBSDs is shown in Figure 24.
Fig. 11. Performance comparison (Scenario II)

Fig. 12. Clustering algorithm (5 clusters)
Fig. 13. Clustering algorithm (4 clusters)

Fig. 14. Clustering algorithm (3 clusters)

Fig. 15. Clustering algorithm (2 clusters)
Fig. 16. Clustering algorithm (1 cluster)

Fig. 17. Power and bandwidth CDF

Fig. 18. Rate per km$^2$
Fig. 19. Clustering algorithm on grid (4 clusters)

Fig. 20. Clustering algorithm on grid (3 clusters)

Fig. 21. Clustering algorithm on grid (2 clusters)
Fig. 22. Clustering algorithm on grid (1 cluster)

Fig. 23. Rate per km$^2$ for regular grid of $I = 9$ CBSDs

Fig. 24. Comparison proportional and equal assignment

Fig. 25. Rate per km$^2$ (comparison proportional and equal assignment)

Fig. 26. Rate per km$^2$ (comparison proportional and equal assignment) for max power 30 dBm
VIII. CONCLUSION

In the paper, we dealt with the efficient allocation of resources (power and frequency) among CBSDs under the control of the coexistence manager. We have compared the performance of the four presented heuristic algorithms with new multi-choice solutions. Based on the achieved results which are projected onto the space of available solutions, we can draw the following conclusions. First, the four heuristic algorithms provide only one, typically suboptimal solution which lays close to the Pareto optimal frontier curve of the solution space. The proposed Multi-Choice Algorithm delivers multiple solutions (all located on the optimal frontier curve), which in turn gives the coexistence manager a new degree of freedom in selecting the best solution based on the chosen selection criteria.

APPENDIX

DERIVATION OF SUM AREA RATE FOR TWO CBSDS

Let us consider a single CBSD utilizing $B_0$ bandwidth and transmitting with power $P_0$ per 5 MHz channel. This results in cell radius $R_0$ according to (1). The formula (18) can be utilized to calculate its sum area rate. Assuming the system is noise limited (interference power in denominator is neglected) and after transferring problem from Cartesian to polar coordinates, i.e., $(x, y) \rightarrow (r, \phi)$, it is obtained

$$S_0 = \int_0^{R_0} \int_0^{2\pi} B_0 \log_2 \left( 1 + \frac{\alpha P_0 r^{-2}}{N} \right) r d\phi dr. \quad (21)$$

Observe that the lower integral limit over the radius is 1 m. As mentioned previously, the FSL propagation model fails for very low distances. A distance of 1 m is reasonable and simplifies the results. Variables $\alpha P_0$ can be replaced in the above formula by utilizing $P_{\text{SEN}} R_0^2$ based on (1) giving

$$S_0 = \int_0^{R_0} \int_0^{2\pi} B_0 \log_2 \left( 1 + \frac{P_{\text{SEN}} R_0^2 r^{-2}}{N} \right) r d\phi dr. \quad (22)$$

While the inner integration is trivial, giving

$$S_0 = \int_0^{R_0} 2\pi B_0 \log_2 \left( 1 + \frac{P_{\text{SEN}} R_0^2 r^{-2}}{N} \right) r dr \quad (23)$$

the outer is calculated using Wolfram alpha tool, giving

$$S_0 = \frac{2\pi B_0}{\ln(4)} \left( R_0^2 \ln \left( \frac{P_{\text{SEN}}}{N} + 1 \right) + \frac{P_{\text{SEN}} R_0^2}{N} \ln \left( \frac{P_{\text{SEN}} R_0^2}{N} + R_0^2 \right) \right) - \ln \left( \frac{P_{\text{SEN}} R_0^2}{N} + 1 \right) - \ln \left( \frac{P_{\text{SEN}} R_0^2}{N} + 1 \right). \quad (24)$$

Let us assume there are two CBSDs sharing bandwidth $B$. If the maximum transmit power is used, cell radius $R_{\text{MAX}}$ is obtained. Assuming that $P_{\text{INT}} = P_{\text{SEN}}$ (as used in simulations section), the whole bandwidth and maximum power can be used by each CBSD if the distance between them is higher than $2R_{\text{MAX}}$. However, for a smaller distance, i.e., $2xR_{\text{MAX}}$ for $x \in (0, 1)$, there are two possible solutions: A) each cell uses an orthogonal band of $B_0 = B/2$ bandwidth, but covers the maximal radius of $R_0 = R_{\text{MAX}}$; B) each cell uses the whole band $B_0 = B$, but has to reduce its radius to $R_0 = xR_{\text{MAX}}$.

The sum rate of each CBSD according to the above formula equals:

$$S_A = \frac{\pi B}{\ln(4)} \left( R_{\text{MAX}}^2 \ln \left( \frac{P_{\text{SEN}}}{N} + 1 \right) - \ln \left( \frac{P_{\text{SEN}} R_{\text{MAX}}^2}{N} + 1 \right) \right) + \frac{P_{\text{SEN}} R_{\text{MAX}}^2}{N} \ln \left( \frac{P_{\text{SEN}} R_{\text{MAX}}^2}{N} + R_{\text{MAX}}^2 \right) - \frac{P_{\text{SEN}} R_{\text{MAX}}^2}{N} \ln \left( \frac{P_{\text{SEN}} R_{\text{MAX}}^2}{N} + 1 \right) \right), \quad (25)$$

and

$$S_B = \frac{2\pi B}{\ln(4)} \left( x^2 R_{\text{MAX}}^2 \ln \left( \frac{P_{\text{SEN}}}{N} + 1 \right) - \ln \left( \frac{P_{\text{SEN}} x^2 R_{\text{MAX}}^2}{N} + 1 \right) \right) + \frac{P_{\text{SEN}} x^2 R_{\text{MAX}}^2}{N} \ln \left( \frac{P_{\text{SEN}} x^2 R_{\text{MAX}}^2}{N} + x^2 R_{\text{MAX}}^2 \right) - \frac{P_{\text{SEN}} x^2 R_{\text{MAX}}^2}{N} \ln \left( \frac{P_{\text{SEN}} x^2 R_{\text{MAX}}^2}{N} + 1 \right). \quad (26)$$

It can be shown that $\lim_{x \to 0} S_B = 0$ and $\lim_{x \to 1} S_B = 2SA$. As such, it is not efficient to use scheme B for closely located cells, i.e., $x \to 0$, but up to 2-fold mean rate increase can be achieved for significantly distanced CBSDs, i.e., $x \to 1$. As $S_B$ is a continuous function, there must be such an $x$ that results in $S_A = S_B$. In typical systems receiver sensitivity is much higher than thermal noise floor, i.e., $P_{\text{SEN}} \gg N$, and the maximal cell radius is much greater than 1 m, i.e., $R_{\text{MAX}} \gg 1$ that allows us to use approximations $\ln \left( \frac{P_{\text{SEN}}}{N} + 1 \right) \approx \ln \left( \frac{P_{\text{SEN}}}{N} \right)$, $\ln \left( \frac{P_{\text{SEN}} R_{\text{MAX}}^2}{N} + 1 \right) \approx \ln \left( \frac{P_{\text{SEN}} R_{\text{MAX}}^2}{N} \right) + \ln \left( x^2 + \frac{P_{\text{SEN}} R_{\text{MAX}}^2}{N} \right) \approx \ln \left( x^2 \right)$. Additionally, the optimal $x$ value is to be close to 1 allowing us to approximate $\ln \left( x^2 \right)$ by its first Taylor series term, i.e., $x^2$. This results in

$$S_A = \frac{\pi B}{\ln(4)} \left( \left( R_{\text{MAX}}^2 + 1 \right) \ln \left( \frac{P_{\text{SEN}}}{N} \right) \right), \quad (27)$$

$$S_B = \frac{2\pi B}{\ln(4)} \left( \left( x^2 R_{\text{MAX}}^2 + 1 \right) \ln \left( \frac{P_{\text{SEN}}}{N} \right) - \ln \left( R_{\text{MAX}}^2 \right) - x^2 + 1 \right). \quad (28)$$

In both $S_B$ and $S_A$ the highest value will have component $R_{\text{MAX}}^2 \ln \left( \frac{P_{\text{SEN}}}{N} \right)$. Omitting all other components, we obtain

$$\frac{S_B}{S_A} \approx \frac{2x^2 R_{\text{MAX}}^2 \ln \left( \frac{P_{\text{SEN}}}{N} \right)}{R_{\text{MAX}}^2 \ln \left( \frac{P_{\text{SEN}}}{N} \right)} = 2x^2. \quad (29)$$

Observe that both mentioned above limits, i.e., $\lim_{x \to 0} S_B = 0$ and $\lim_{x \to 1} S_B = 2SA$, are met by this approximation. The equal rate of both schemes is obtained for $x = \sqrt{2}$, meaning a single cluster of cells is advantageous for a cell radius higher than $0.71R_{\text{MAX}}$. The accuracy of this approximation has been confirmed for various values of parameters $R_{\text{MAX}}$ and $\frac{P_{\text{SEN}}}{N}$ in Figure 27. Observe that this result is valid for the case of two, noise-limited cells.

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